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Meta Math

Informational technologies and electronic tutoring for preparation for mathematical competitions

M.I. Malkin
E.V. Malkina
V.I. Shvetsov

Experience

M.I. Malkin
Professor of UNN, Chairman of the Nizhny Novgorod region jury in Mathematics for All-Russian Olympiad and for the inter-regional Olympiad “Future researchers – the future of science”, author of Olympiad problems.

V.I. Shvetsov
Professor of UNN, the organizer of the inter-regional Olympiad “Future researchers – the future of science”, expert in the field of information technology and e-learning. Author of the first in UNN on-line course

E.V. Malkina
Professor of UNN, a specialist in the field of information technology and e-learning. Author of several on-line courses, e-learning site administrator UNN

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Mathematical Olympiads supported by Lobachevcky University

- All-Russian Olympiad in Mathematics, the regional Tour
- Inter-regional Olympiad "Future researchers - the future of science"
- International Mathematical Olympiad “Tournament of Towns”

Second (municipal) stage of all-Russain Olymiad in Mathematics, 2016/17 academic year.

Each test consists of 5 problems, each problem costs 7 points.

Scored more than 4 points

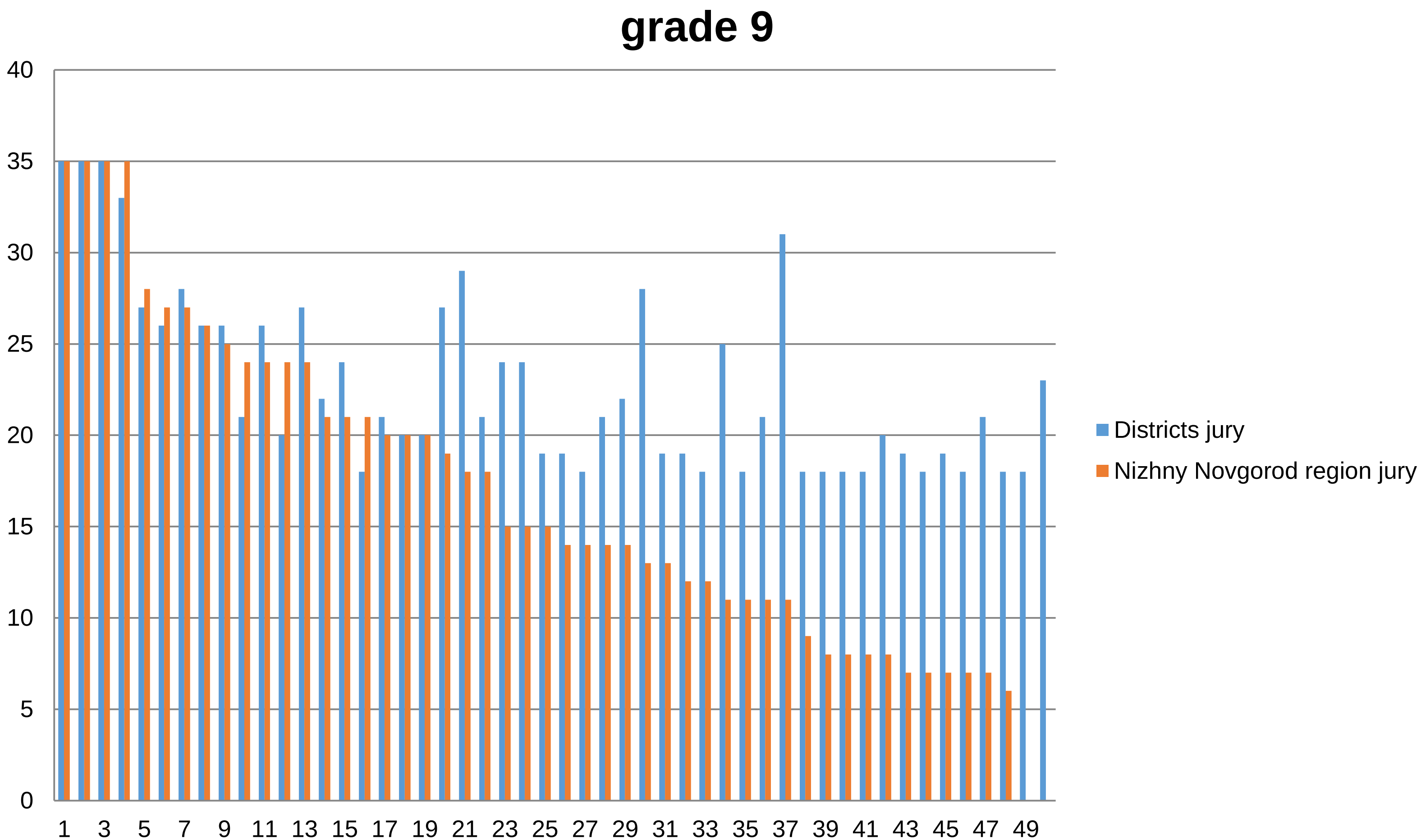
	All participants	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5
Grade 9	**50**	**31**	**30**	**26**	**5**	**16**
Grade 10	**59**	**46**	**54**	**27**	**3**	**38**
Grade 11	**121**	**96**	**62**	**58**	**57**	**42**

Problems

Lack of experience of school teachers in specific Olympiad topics is particularly evident while inspection of evaluations made by jury of different districts during the municipal stage of the All-Russian Olympiad in Mathematics. After rechecking by the regional jury, often there are cases where it appears that the good works, for whatever reason, were not evaluated and, conversely, overvalued weaker works.

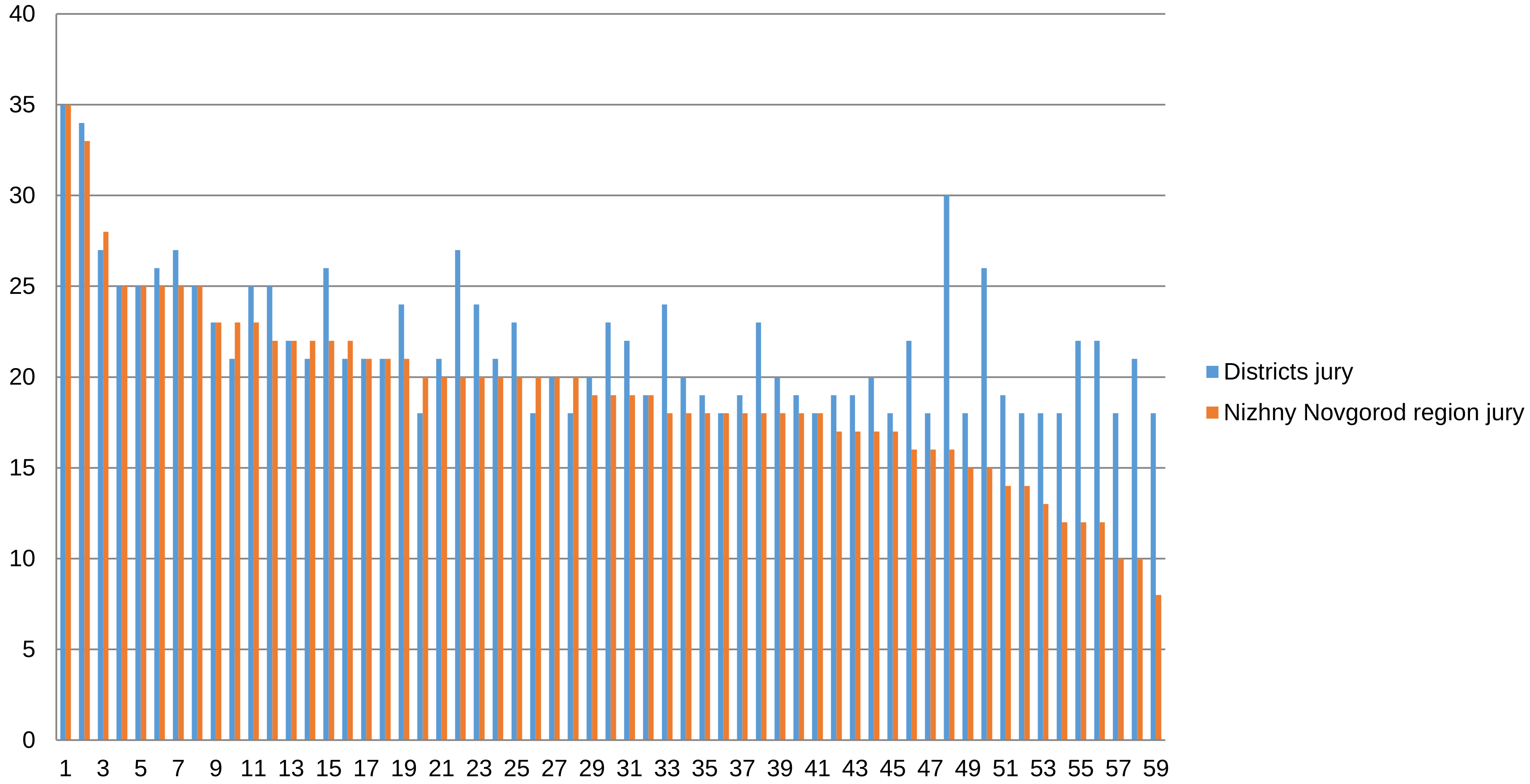
- Even having written solutions of Olympiad problems, not all school teachers can properly assess the student's solution.
- Due to the strong quota, participants can not get to the next round if they have not been properly evaluated.

Comparison of evaluations by districts jury and regional jury

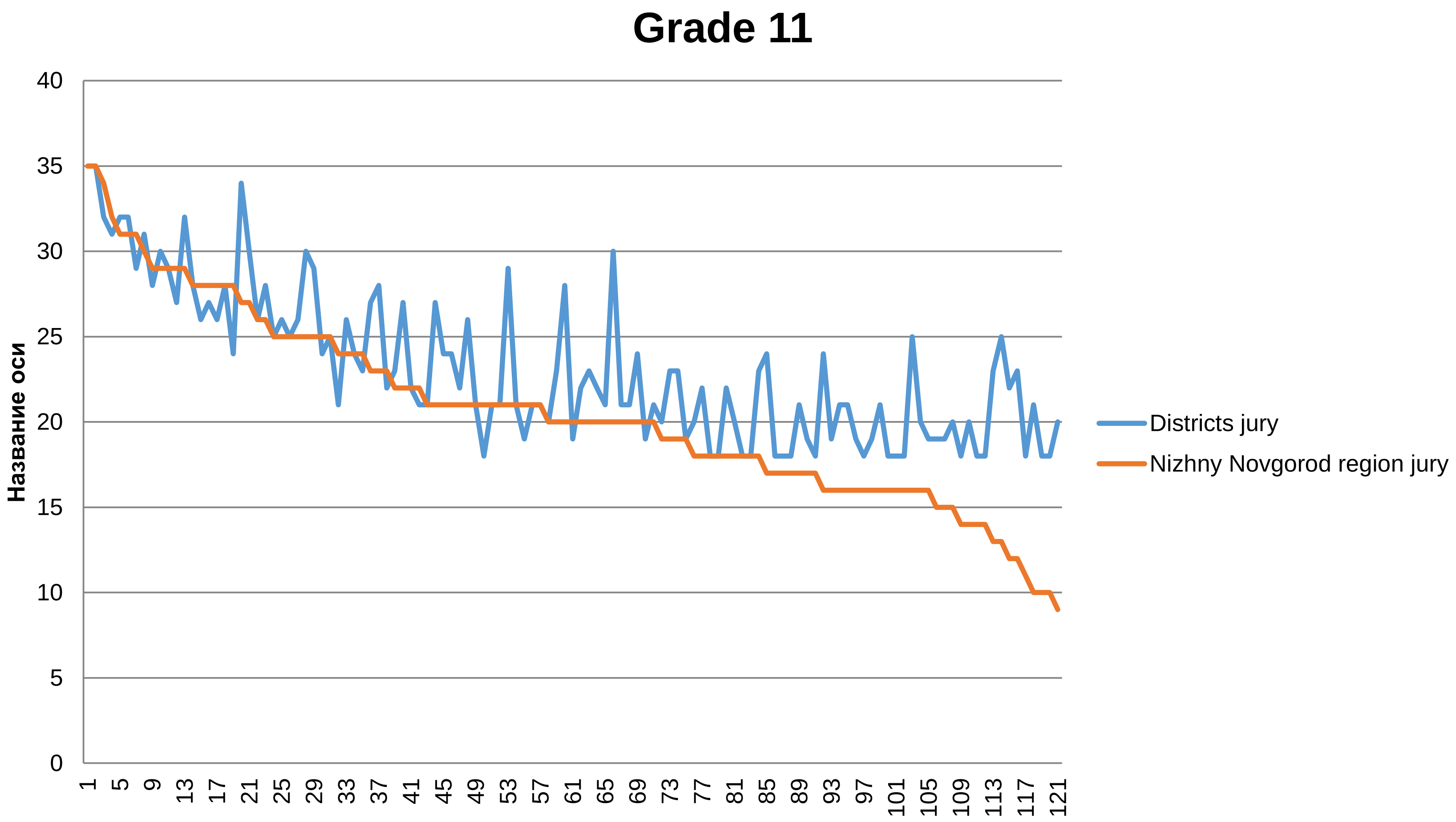


Comparison of evaluations by districts jury and regional jury

Grade 10



Comparison of evaluations by districts jury and regional jury



Problems

In the courses for teachers training organized at the Department of Mathematics of the Nizhny Novgorod Institute of Education Development (NIRO), some attention is paid to the Mathematical Olympiads, but an insufficient amount of lectures in the program and a limited number of trained school teachers can not solve the problem.

Typically, students teams taking part in competitions at various levels, including international, held regular training, where they studied special methods, techniques solve problems, get in-depth knowledge of topics of elementary math and knowledge that go beyond elementary mathematics.

Each year, educational principals ask the specialists in Math Olympiads to train the students in their particular schools.

Decision

In our opinion, there can help
Creating massive open on-line course
"Mathematical Olympiads",
to train both students and school teachers.

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The content of the MEP "Mathematics Olympiad»

The course will include some areas of mathematics that are absent in the school program (in number theory, the theory of polynomials, combinatorics, elements of mathematical analysis, complex numbers, classical and combinatorial geometry), as well as examples of solving Olympiad problems of varying complexity.
The course can be extended.

Used technologies

As the main system for development of the training course for solving Olympiad problems, a popular and proven Modular Object-Oriented Dynamic Learning Moodle Environment has been selected.

It is a widespread and popular system used in e-learning system of the Nizhny Novgorod University since 2009.

Many settings in the system allow one to adapt it to different needs. Since this environment uses Web technologies, with the help of hyperlinks it can be used in the system of various online resources.

In addition, specially designed modules are included in the Moodle, allow one to use and synchronize account information from external specialized training systems, use custom illustrations and tests as elements of training.

Used technologies

For the external training system we use Math-Bridge.
Math-Bridge is an intelligent system of math learning, based on the system of artificial intelligence.
In this system, there is a knowledge base of educational material, which is supported by a cross-cultural and multilingual access.
All training material in Math-Bridge is divided into atomic objects, called the elements of knowledge («knowledge items»), of which training facilities are being built. As the basic model of Math-Bridge domain uses an ontology of abstract mathematical knowledge.
Courses in this system built using collections of teaching material covering various topics of mathematics for middle and high school..

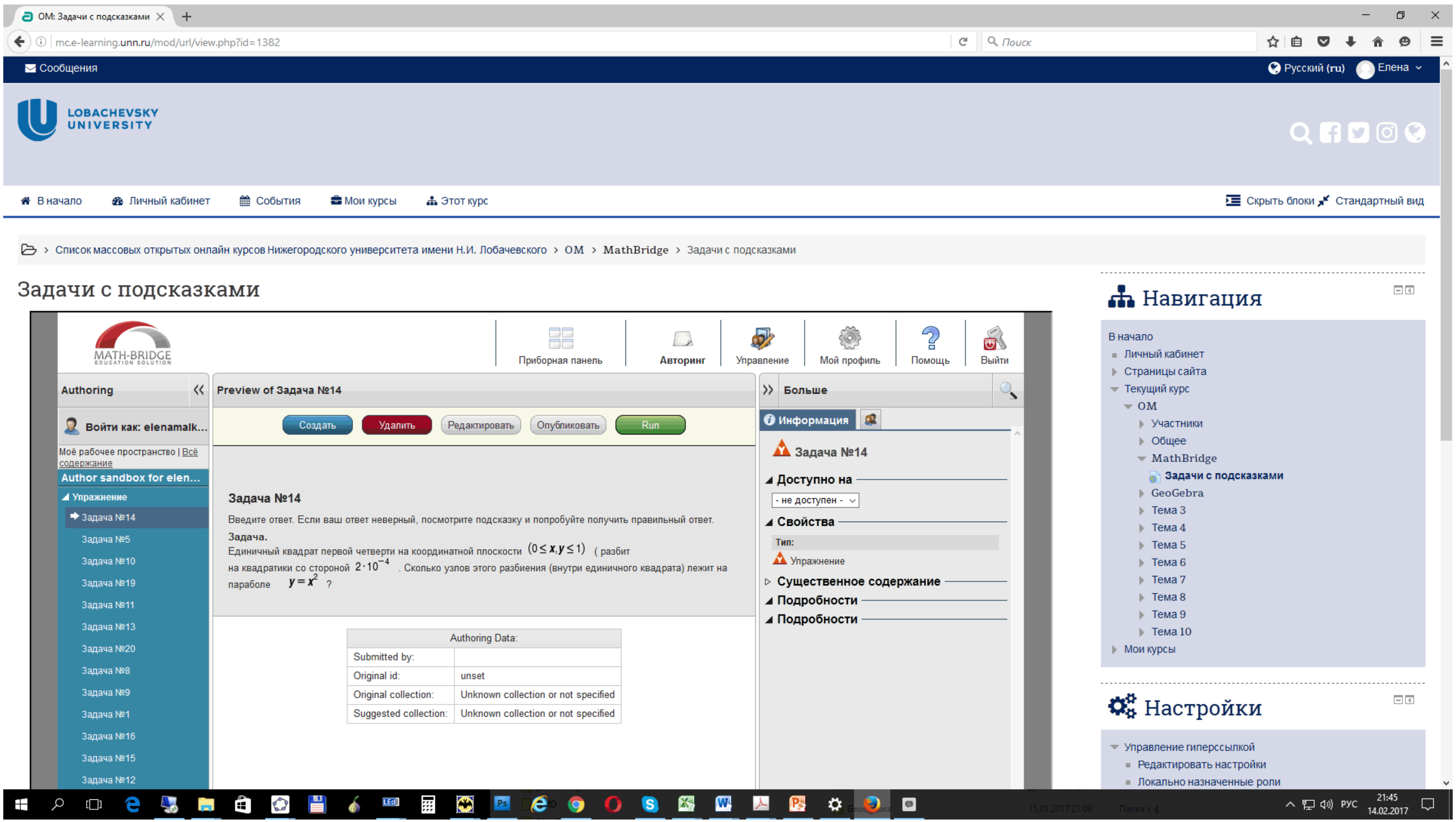
MATH-BRIDGE
EDUCATION SOLUTION

Used technologies

Math-Bridge supports large educational experience: it includes a large amount of educational material with sufficiently detailed visual computer illustrations for mathematical models and methods, control tasks, involving selection of not only one among the given answers, instead it proposes a training sequence of steps leading to the decision (stepper control system). At the same time, it includes various types of learning objects: definitions, theorems, proofs, examples and interactive exercises.

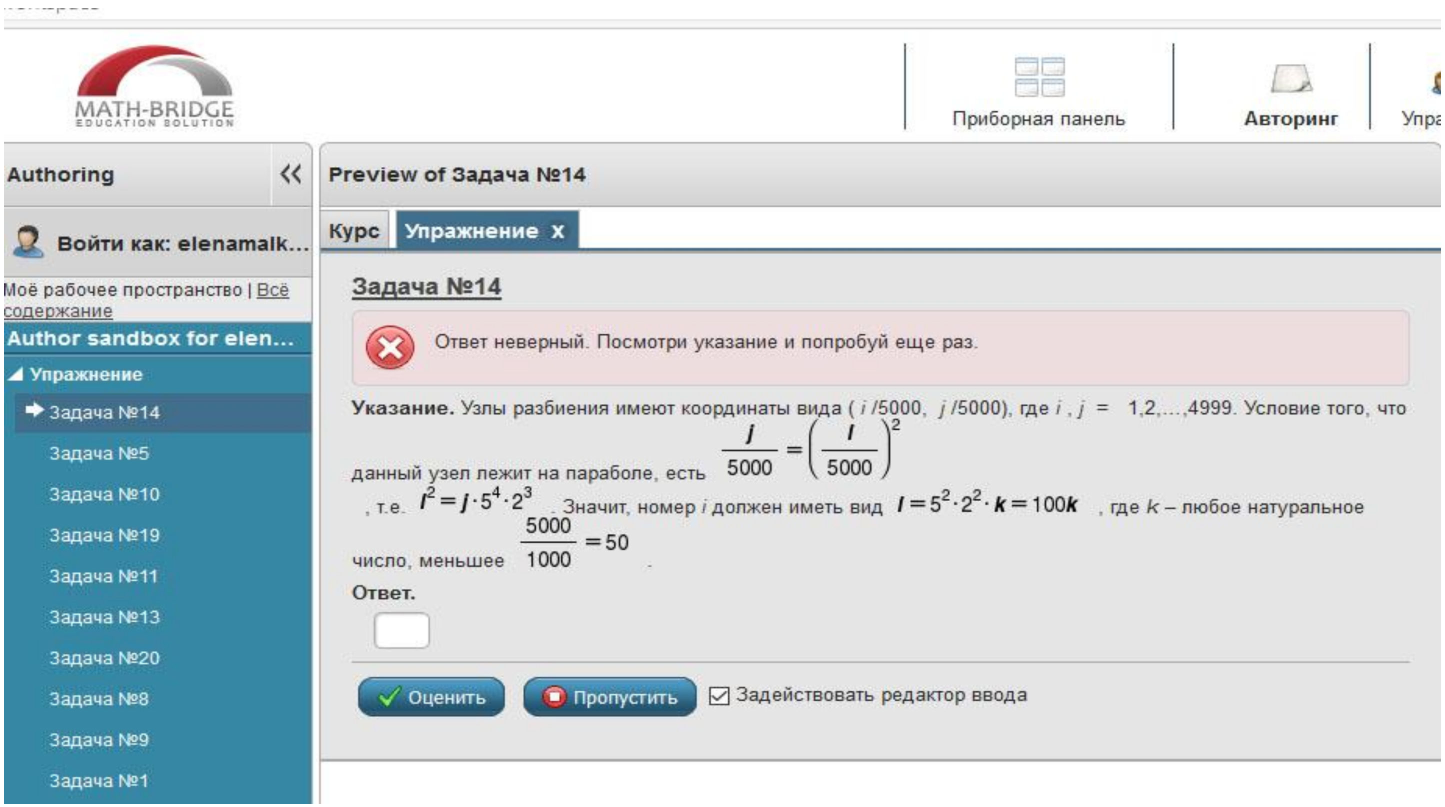
Math-Bridge users can choose one of the many predefined courses or dynamically generate math courses, adapted to the purpose of the particular student, his/her preferences, capabilities and current knowledge.

Using Math-Bridge in Moodle environment



Used technologies

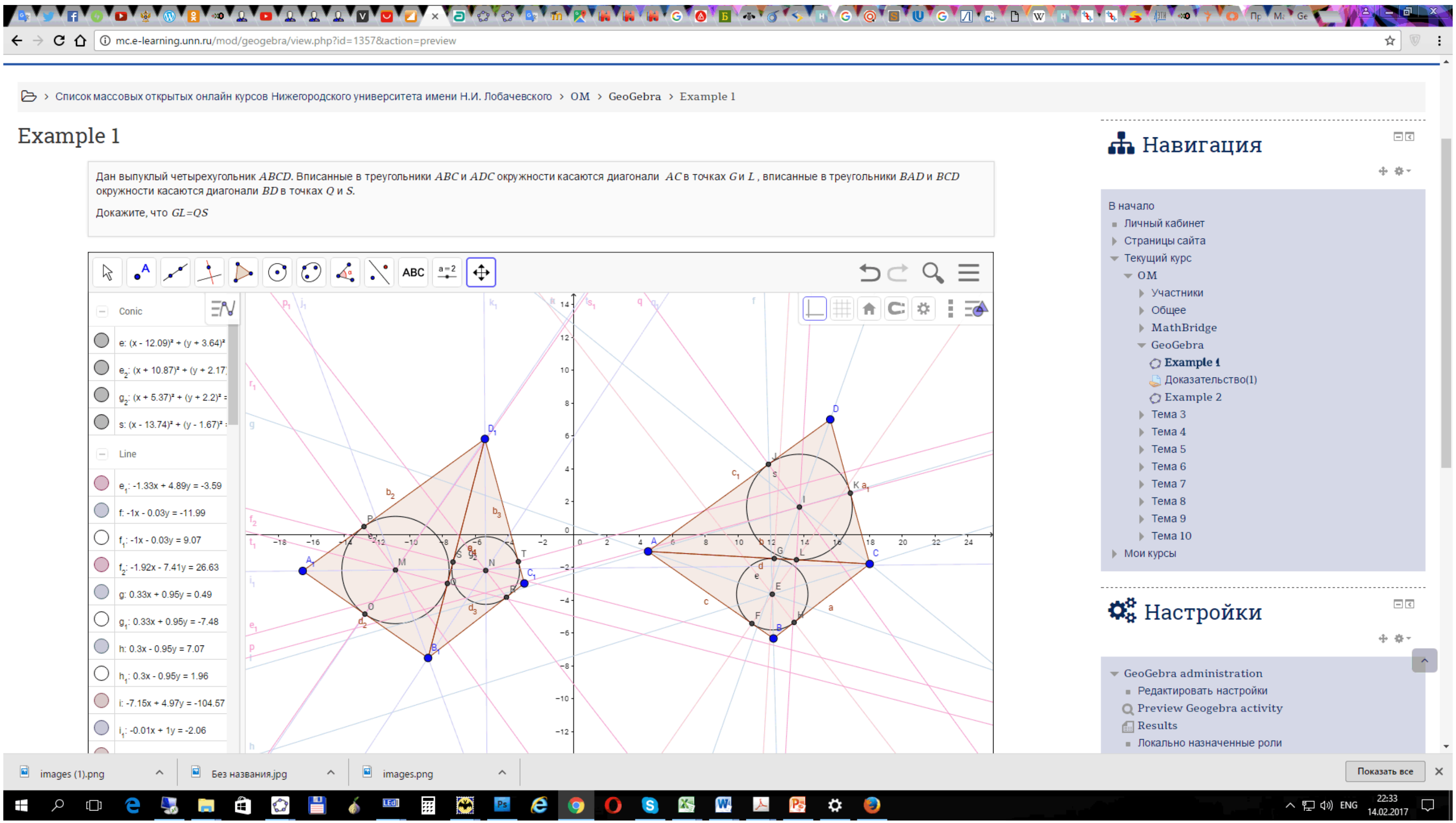
When forming the Math-Bridge tasks, one can ask for numerical or analytical solutions, and in the case of incorrect answer, the system may offer tips and give the student the opportunity to solve the problem once again.



Used technologies

DYNAMIC GEOMETRICAL Environment GeoGebra
GeoGebra is a free, cross-platform dynamic mathematical program for all levels of education, which includes geometry, algebra, tables, graphs, statistics and arithmetics.
In addition, the program has high possibilities to work with functions (graphing, calculating roots, extrema, integral, and so on.) Due to the built-in command language it allows to manage with geometric constructions.
The program has been written by Marcus Hohenvarterom in Java. It runs on a large number of operating systems. It was translated into 39 languages and is being actively developed.
The program allows one to create visual representations of various mathematical objects that is relevant in the study of mathematics.

Using GeoGebra in Moodle environment

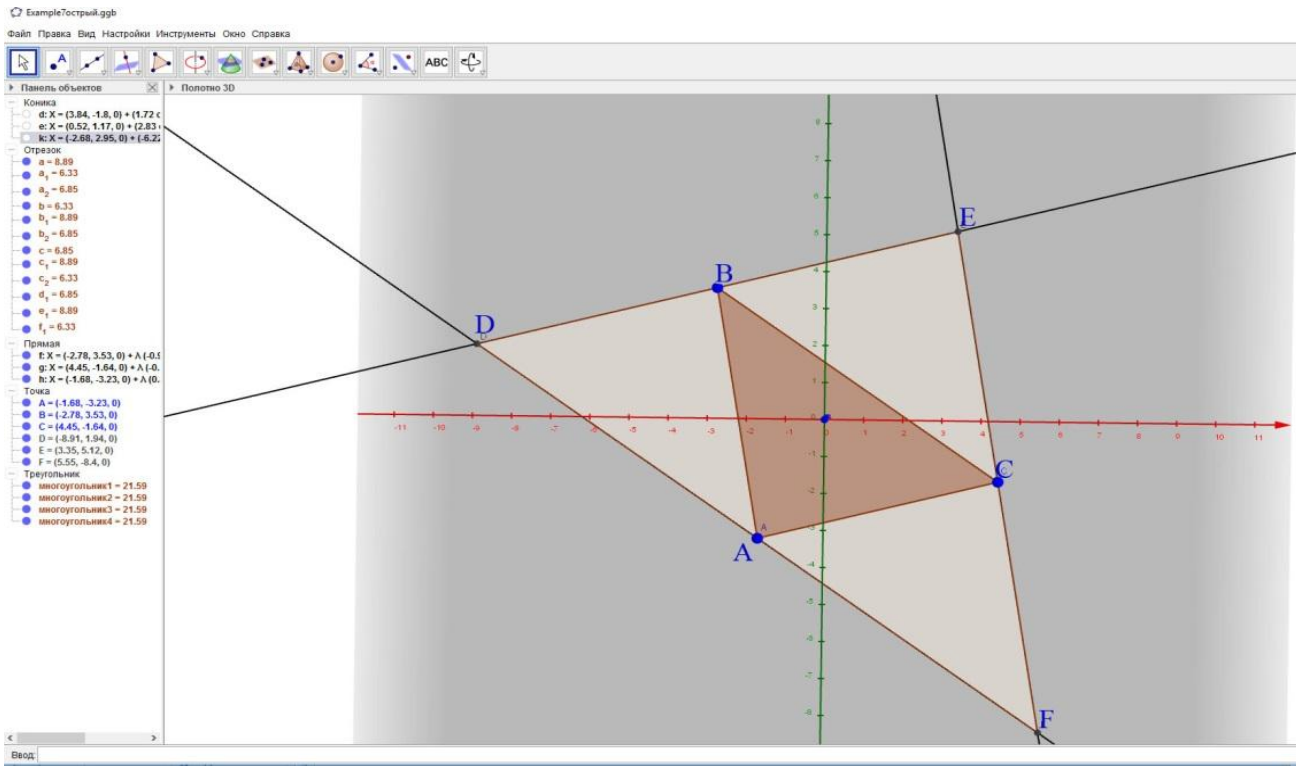


Using GeoGebra

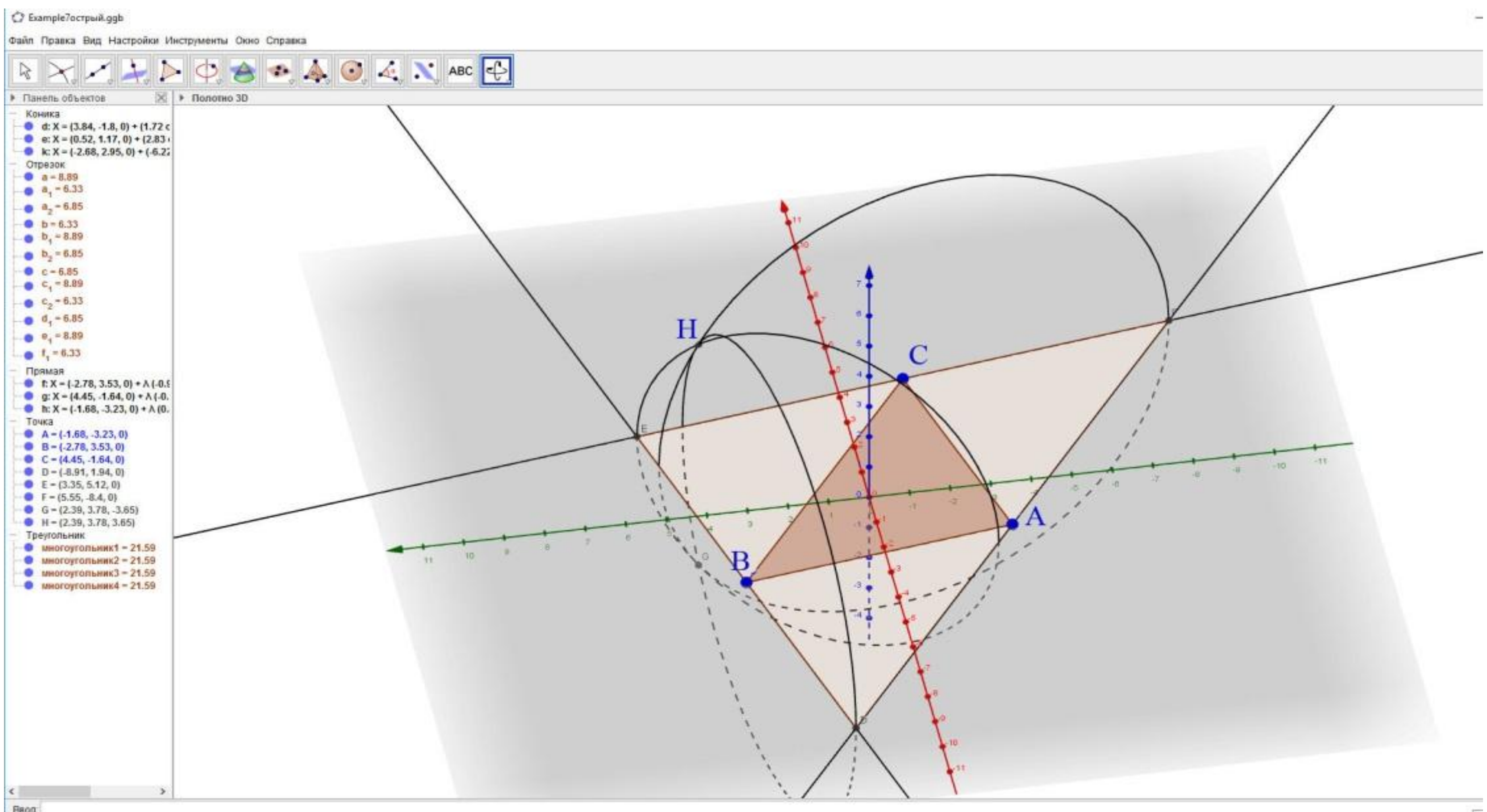
The use of GeoGebra system can help the teacher to draw attention to theoretical knowledge of mathematics and to the evidence of its importance, especially in those situations when the first intuitions are incorrect or are valid only under certain conditions, and these conditions should determine analytically and illustrate them then using GeoGebra. As an example, consider the following problem which was proposed to the municipal stage of All-Russian Olympiad in 2011-2012 academic year:

Given an acute-angled triangle ABC, prove that there exists a tetrahedron, all of whose faces are triangles, equal to the triangle ABC.

Most of the participants solved this problem based on the practical notion that is the plain scan of a tetrahedron, and how to glue the tetrahedron using the scan. Arguments of participants were as follows: Consider a triangle Δ*A´B´C´* for which sides of the triangle *ΔABC* are average lines. (Formally, *ΔABC and ΔA´B´C´ are* homothetic relative to the intersection point of the medians with coefficient -2).

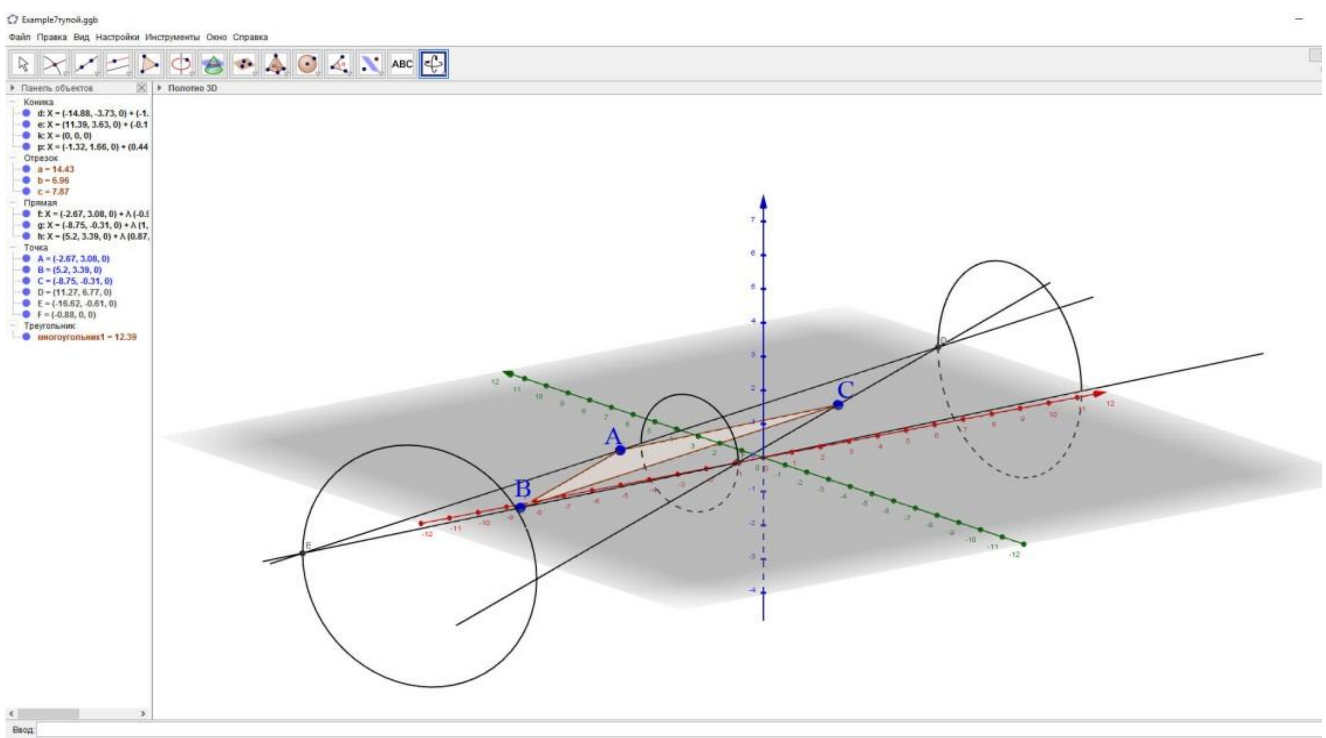


Then, turning in the space the triangles *ABC´, BCA´* and *ACB´* around the sides of the triangle *ABC* as the fold lines, as a result of the connection (and bonding) relevant parties obtain the desired tetrahedron.. GeoGebra system allows us to trace this process and find a point of connection faces *H.* In the figure we changed the point of view of the previous figure, to see it better.



Students are not sufficiently serious to the evidence, and teachers need to pay attention to it, citing various examples where such neglect leads to serious errors. In this problem, you can use GeoGebra to demonstrate that this method works in case of an acute-angled triangle and does not work for obtuse triangle.

Indeed, for obtuse triangles vertices *A´, B´, C´* while rotating form non-intersecting circles.

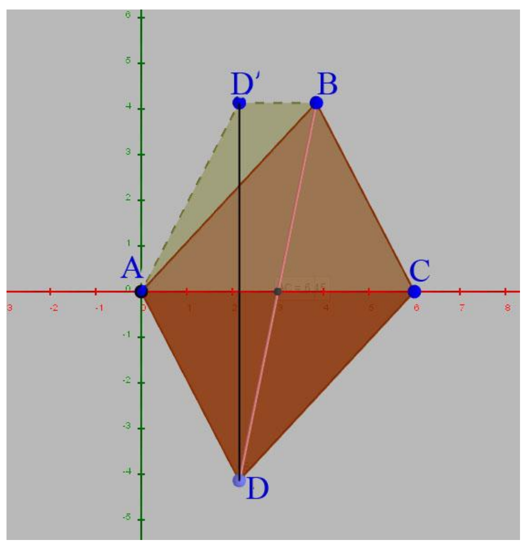


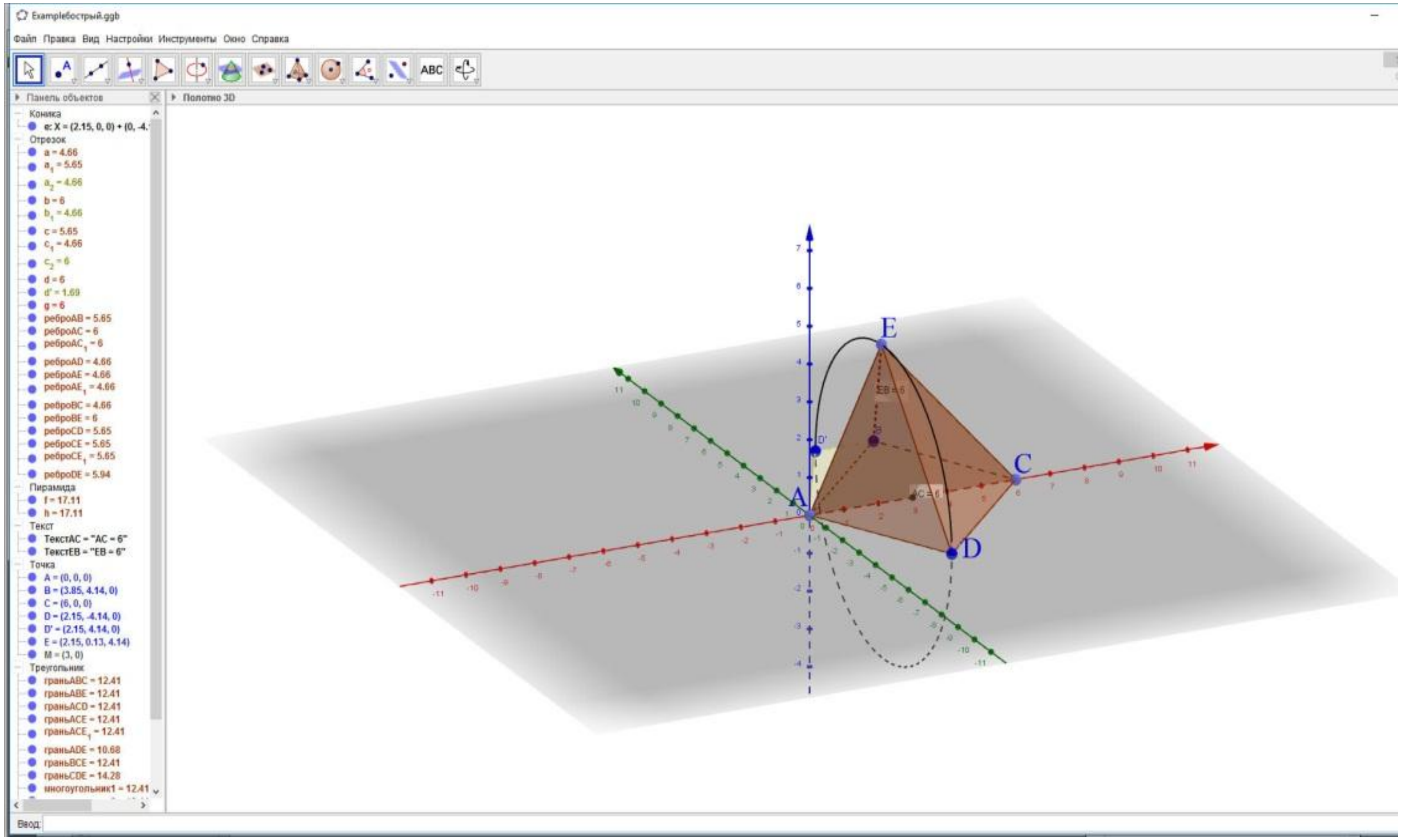
After that you can lead to a rigorous proof, which becomes the most prepared and motivated. To do this, we first prove the lemma:

If in the triangle *ABC, the angle B is acute, then the median drawn from the vertex B, is lager than half of any of the sides of* ΔABC.

Proof: Let *M* be the midpoint of side *AC*, *D* the point symmetric of B with respect to M. Then *ABCD* is parallelogram, where M is the point of intersection of the diagonals. Then the angle *BAD* is obtuse, since it is equal to $\angle BAC + \angle BCA = 180° - \angle B$. Therefore, *A* and *C* lie inside the circle with center *M* of radius *BM*. Then the segments *AB*, *BC* and *AC* are smaller than the diameter *BD* (= 2*BM*) of this circle.

Let the point D' be symmetric to the point D with respect to the line AC. Then $AD'BC$ is an isosceles trapezoid. Since the angle ABC is acute, $BD>AC$ *(by the lemma)*, and since the angles *BAC and BCA* are acute, then $D'B<AC$. If we rotate $\Delta\ ADC$ in the space around the fixed line AC, then the point D will move in a circle. Denote its position at the time t by D_t. The distance from D_t to B will change from $DB>AC$ to $D'B<AC$. Therefore there is a moment when $D_t B = AC$. At this moment, all the faces of the tetrahedron $ABCD_t$ are equal to the triangle ABC.





The rotation of Δ *ADC* in the space around the fixed line *AC*

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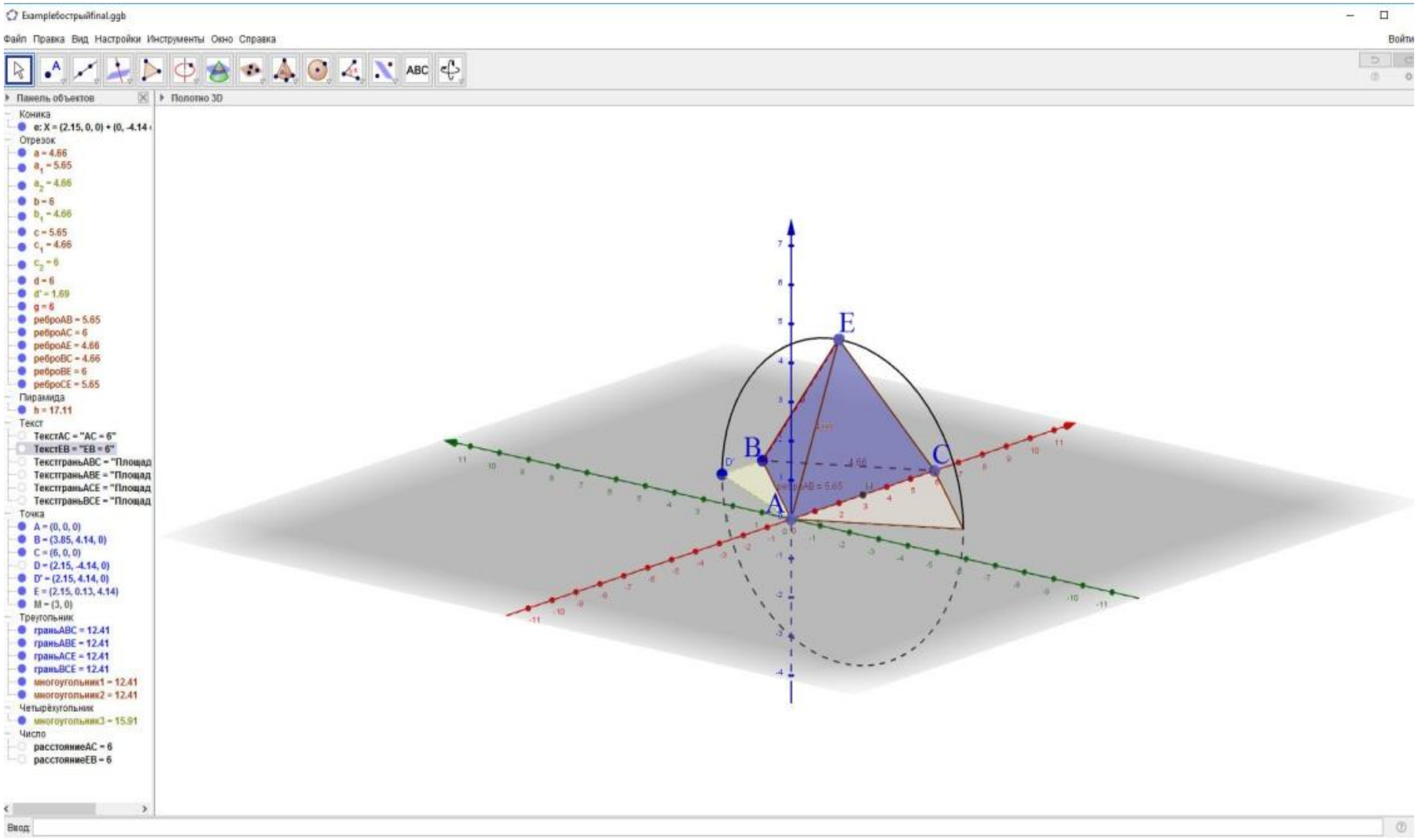

All faces of the tetrahedron ***ABCE(D_t)*** are equal to the triangle ***ABC***

Conclusion

Creation of e-learning course use complex combinations of multiple software systems Moodle, Math-Bridge, GeoGebra et al. Using these technologies in Olympiad Mathematics, will help to improve the skills in solving Olympiad problems for both students and school teachers.

MEP "Mathematical Olympiads" should become a point of attraction for talented students and a place for their communications